

FINAL TERM EXAMINATION

Fall 2009

Calculus & Analytical Geometry-I

Question No: 1 (Marks: 1) - Please choose one

$$y = \frac{x^2}{2}$$

Let $y = \frac{x^2}{2}$. Find average rate of change of y with respect to x over the interval $[3, 4]$

☐ $\frac{25}{2}$



☒ $\frac{7}{2}$



☐ $\frac{25}{14}$



☐ $\frac{7}{14}$



Question No: 2 (Marks: 1) - Please choose one

If $2x - y = -3$ then $\frac{dy}{dx} =$

☒ 2

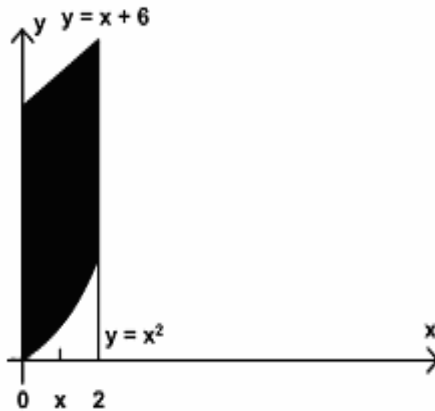
☐ -2

☐ 0

☐ -3

Question No: 3 (Marks: 1) - Please choose one

In the following figure, the area bounded on the sides by the lines are :



- ▶ $x = 0$
- ▶ $x = 2$
- ▶ $x = 0 \text{ and } x = 2$
- ▶ $x = 6$

Question No: 4 (Marks: 1) - Please choose one

What is the sum of following series?

$1 + 2 + 3 + 4 + ______ + n$

- ▶ $\frac{n+1}{2}$
- ▶ $\frac{(n+1)(n+2)}{2}$
- ▶ $\frac{n(n+2)}{2}$
- ▶ $\frac{n(n+1)}{2}$
- ▶ $\frac{n(n+1)}{2}$

Question No: 5 (Marks: 1) - Please choose one

Let f is a smooth function on $[0, 3]$. What will be the arc length L of the curve $y = f(x)$ from

$x = 0$ to $x = 3$?

- ▶ $L = \int_0^3 \sqrt{1 + [f(x)]^2} dy$

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}$$



$$L = \int_0^3 \sqrt{1 + [f'(x)]^2} dy$$



$$L = \int_0^3 \sqrt{1 + [f'(x)]^2} dx$$



Question No: 6 (Marks: 1) - Please choose one

The PYTHAGORAS theorem describes the relationship between the sides of▶ **Right angle triangle**

▶ **Right angle triangle**

▶ Isoceleous triangle

▶ Equilateral triangle

Question No: 7 (Marks: 1) - Please choose one

Which operation can not be applied on the functions?

▶ Subtraction

▶ **Cross product**

▶ Addition

▶ Composition

Question No: 8 (Marks: 1) - Please choose one

The graph of the equation $y = x^2 - 4x + 5$ will represent

▶ **Parabola**

▶ Straight line

▶ Two straight lines

▶ Ellipse

Question No: 9 (Marks: 1) - Please choose one

Polynomials are always functions

▶ **Continuous**

▶ Discontinuous

Question No: 10 (Marks: 1) - Please choose one

The $\tan(x)$ is discontinuous at the points where

- ▶ $\cos(x) = 0$
- ▶ $\sin(x) = 0$
- ▶ **$\tan(x) = 0$**

Question No: 11 (Marks: 1) - Please choose one

A differentiable function must be differentiable on the interval

- $(-\infty, \infty)$
- ▶ $(0, \infty)$
- ▶ $(-\infty, \infty)$
- ▶ (a, ∞) where a is any negative integer

Question No: 12 (Marks: 1) - Please choose one

Let $y = (x^3 + 2x)^{37}$. Which of the following is correct?

$$\frac{dy}{dx} = (37)(x^3 + 2x)^{36}$$

▶

$$\frac{dy}{dx} = 111x^2(x^3 + 2x)^{36}$$

▶

$$\frac{dy}{dx} = (111x^2 + 74)(x^3 + 2x)^{36}$$

▶

$$\frac{dy}{dx} = (111x^2 + 74)(x^3 + 2x)^{38}$$

▶

Question No: 13 (Marks: 1) - Please choose one

$$\int \frac{3x^2 + 4x + 1}{x^3 + 2x^2 + x - 3} dx$$

Consider the indefinite integral

Let $t = x^3 + 2x^2 + x - 3$

Is the following substitution correct?

$$\int \frac{3x^2 + 4x + 1}{x^3 + 2x^2 + x - 3} dx = \int \frac{1}{t} dt$$

- ▶ Yes
- ▶ **No**

Question No: 14 (Marks: 1) - Please choose one

$\log_b ac = \underline{\hspace{2cm}}$

▶ $\log_b a + \log_b c$

▶ $\log_b a - \log_b c$

▶ $\frac{\log_b a}{\log_b c}$

▶ $(\log_b a)(\log_b c)$

▶

Question No: 15 (Marks: 1) - Please choose one

If a function has an extreme value (either a maximum or a minimum) on an open interval (a,b), then the extreme value occurs at of f

▶ First point

▶ Mid point

▶ **Critical point**

▶ End point

Question No: 16 (Marks: 1) - Please choose one

The Mean Value Theorem states that "Let function f be differentiable on (a,b) and continuous on $[a, b]$, then there exist at least one point c in (a,b) where"

▶ $f'(c) = \frac{f(b) - f(a)}{b - a}$

▶ $f(c) = \frac{f(b) - f(a)}{b - a}$

▶

▶ $f(c) = \frac{f(a) - f(b)}{b - a}$

▶

▶ $f'(c) = \frac{f(a) - f(b)}{b - a}$

▶

Question No: 17 (Marks: 1) - Please choose one

$$\frac{d}{dx}[F(x)] = f(x)$$

If there is some function F such that then any function of the form $F(x) + C$ is of $f(x)$

- ▶ Derivative
- ▶ **Antiderivative**
- ▶ Slope
- ▶ Maximum value

Question No: 18 (Marks: 1) - Please choose one

$$\sum_{k=1}^n f(x_k^*) \Delta x_k$$

The sum is known as:

- ▶ **Riemann Sum**
- ▶ General Sum
- ▶ Integral Sum
- ▶ Geometric Sum

Question No: 19 (Marks: 1) - Please choose one

$$\int_0^{\frac{\pi}{2}} \cos u \, du$$

If , then which of the following is true?

- ▶ 2
- ▶ **1**
- ▶ 0
- ▶ -1

Question No: 20 (Marks: 1) - Please choose one

$$\int_0^{\pi} \sin u \, du$$

If , then which of the following is true?

- ▶ 1
- ▶ 2
- ▶ **0**
- ▶ -1

Question No: 21 (Marks: 1) - Please choose one

$$\frac{d}{dx}[F(x)] = f(x)$$

If there is some function F such that then antiderivatives of $f(x)$ are $F(x) + C$. What does C represents?

- ▶ Polynomial
- ▶ **Constant**
- ▶ Dependent Variable
- ▶ Independent Variable

Question No: 22 (Marks: 1) - Please choose one

If f and g are continues function on an interval $[a, b]$
and $f(x) \geq g(x)$ for $a \leq x \leq b$, then area is bounded by the lines parallel to:

- ▶ X -axis
- ▶ Y-axis
- ▶ Both X -axis and Y-axis

Question No: 23 (Marks: 1) - Please choose one

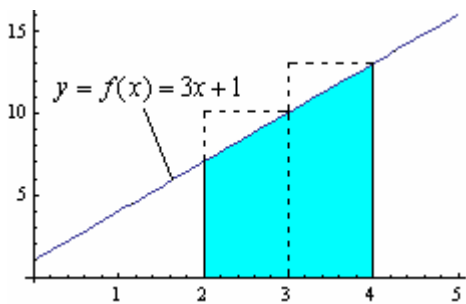
$\int_1^{2/3} dx =$ _____

- ▶ $-\frac{1}{3}$
- ▶ 0
- ▶ $\frac{1}{3}$
- ▶ $\frac{2}{3}$

Question No: 24 (Marks: 1) - Please choose one

$\int_0^2 x \, dx =$ _____

- ▶ 2
- ▶ 0
- ▶ 2
- ▶ -2
- ▶ $\frac{x^2}{2}$

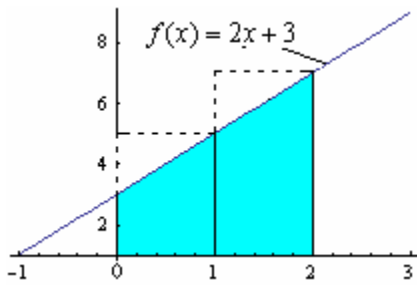


Question No: 25 (Marks: 1) - Please choose one

Which of the following is approximate area of the shaded region by taking x_1^* and x_2^* as left endpoint of equal-length subintervals?

- ▶ 17
- ▶ 20
- ▶ 23
- ▶ 26

Question No: 26 (Marks: 1) - Please choose one



Which of the following is approximate area of the shaded region by taking x_1^* and x_2^* as right endpoint of equal-length subintervals?

- ▶ 8
- ▶ 10

- ▶ 12
- ▶ 14

Question No: 27 (Marks: 1) - Please choose one

What is the length of each sub-interval, if the interval $[1,3]$ is divided into n sub-intervals of equal length?

- ▶ $\frac{1}{n}$
- ▶ $\frac{2}{n}$
- ▶ $\frac{3}{n}$
- ▶ $\frac{4}{n}$
- ▶

Question No: 28 (Marks: 1) - Please choose one

Evaluate

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = \text{-----}$$

- ▶ 4
- ▶ 2
- ▶ 1
- ▶ ∞

Question No: 29 (Marks: 1) - Please choose one

$$\left\{ \frac{1}{2^n} \right\}_1$$

represents the sequence:

$$\frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \dots$$

- ▶

▶ $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots$

▶ $0, 1, \frac{1}{2}, \frac{1}{4}, \dots$

▶ $0, 1, 2, 3, \dots$

Question No: 30 (Marks: 1) - Please choose one

For a sequence $\{a_n\}$ if the difference between successive terms $a_{n+1} - a_n \leq 0$ then the sequence is known as:

- ▶ Increasing
- ▶ Decreasing
- ▶ Nondecreasing
- ▶ **Nonincreasing**

Question No: 31 (Marks: 1) - Please choose one

For a sequence $\{a_n\}$ if the ratio of successive terms $\frac{a_{n+1}}{a_n} > 1$ then the sequence is known as:

- ▶ **Increasing**
- ▶ Decreasing
- ▶ Nondecreasing
- ▶ Nonincreasing

Question No: 32 (Marks: 1) - Please choose one

If the partial sum of a series is finite then the series will/will be:

- ▶ Divergent
- ▶ **Convergent**
- ▶ Give no information

Question No: 33 (Marks: 1) - Please choose one

If the geometric series $a + ar + ar^2 + ar^3 + \dots + ar^{k-1} + \dots$ where $(a \neq 0)$, $|r| < 1$ then which of the following is true for the given series?

- ▶ **Converges**
- ▶ Diverges
- ▶ Gives no information

Question No: 34 (Marks: 1) - Please choose one

If $\rho = \lim_{k \rightarrow +\infty} \sqrt[k]{u_k}$ where $\rho > 1$ then the series $\sum u_k$ with positive terms will /will be.....?

- ▶ **Convergent**

- ▶ Divergent
- ▶ Give no information

Question No: 35 (Marks: 1) - Please choose one

$$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{k}$$

Which of the following is true for the series ?

- ▶ Arithmetic Series
- ▶ Geometric Series
- ▶ **Alternating Harmonic Series**
- ▶ Harmonic Series

Question No: 36 (Marks: 1) - Please choose one

.....is the special case of Tylor's theorem.

▶ **Roll's Theorem**

- ▶ Picard's Method
- ▶ Integration
- ▶ Maclaurin's Theorem

Question No: 37 (Marks: 1) - Please choose one

If f is integrable on a closed interval containing the four points a, b, c and d then

$$\int_a^d f(x) dx = \underline{\hspace{2cm}}$$

▶ $\int_a^b f(x) dx + \int_b^c f(x) dx + \int_c^d f(x) dx$

▶ $\int_a^b f(x) dx + \int_c^d f(x) dx$

▶ $\int_a^c f(x) dx + \int_b^d f(x) dx$

▶ $\int_a^d f(x) dx$

▶

Question No: 38 (Marks: 1) - Please choose one

Suppose f and g are integrable functions on $[a, b]$ and c is a constant, then

$$\int_a^b c [f(x) + g(x)] dx = \underline{\hspace{2cm}}$$

$$\int_a^b f(cx)dx + \int_a^b g(cx)dx$$



$$\int_a^b f(x) dx + \int_a^b g(x) dx$$



$$c \int_a^b f(x)dx + c \int_a^b g(x)dx$$



▶ 0

Question No: 39 (Marks: 1) - Please choose one

What is the difference between the values of the

$$\int_a^b f(x)dx \text{ and } \int_a^b f(t)dt$$

integrals ?

- ▶ Differ by b-a
- ▶ Differ by a-b
- ▶ **No difference**
- ▶ Differ by b+a

Question No: 40 (Marks: 1) - Please choose one

$$\int_{-1}^2 f(x) dx = 5 \quad \int_{-1}^2 g(x) dx = -3$$

If and then which of the following is value

$$\int_{-1}^2 [f(x) + 2g(x)] dx$$

of ?

- ▶ **-1**
- ▶ -8
- ▶ 2
- ▶ 11

Question No: 41 (Marks: 2)

$$\frac{1}{1} + \frac{1}{8} + \frac{1}{27} + \dots + \frac{1}{1000}$$

Express the sum in sigma notation.

$$\sum_{n=1}^{10} (1/n^3)$$

Question No: 42 (Marks: 2)

Only write down the Maclaurin series for e^x

Question No: 43 (Marks: 2)

Evaluate the following integral:

$$\int_1^4 \sqrt{x} \, dx$$

$$\int_1^4 \sqrt{x} \, dx$$

$$= \int_1^4 \sqrt{x} \cdot 1 \, dx$$

$$= x\sqrt{x} + \int_1^4 1/\sqrt{x} \cdot 1 \, dx$$

Question No: 44 (Marks: 3)

Evaluate the following sum:

$$\sum_{k=1}^6 (k^2 - 5)$$

$$= -4 - 1 + 4 + 11 + 20 + 31 = 61$$

Question No: 45 (Marks: 3)

Find a definite integral indicating the area enclosed by the curves $y = x^2$, $x > 0$ and bounded on the sides by the lines $y = 1$ and $y = 4$. But do not evaluate the integral.

Question No: 46 (Marks: 3)

$$a_n = \left\{ \frac{3}{n^2} \right\}_{n=5}^{\infty}$$

Determine whether the following sequence is strictly monotone or not.If your answer is yes or no, then give reason .

Yes the sequence is strictly monotone because the denominator is increasing

Question No: 47 (Marks: 5)

The region bounded by the y -axis, the graph of the equation $x = y^{\frac{3}{2}}$ and the line $y = 2$ is revolved about y -axis. Find the volume of the resulting solid.

Question No: 48 (Marks: 5)

Compute the following sum:

$$\sum_{i=1}^n (4i^2 - i) = (4(1)^2 - 1) + (4(2)^2 - 2) + (4(3)^2 - 3) + (4(4)^2 - 4) \dots\dots\dots$$

$$= 3 + 14 + 33 + 60 \dots\dots\dots$$

Question No: 49 (Marks: 5)

Use L'Hopital's rule to evaluate the limit

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \sin x}{1 + \cos 2x}$$

$$\lim_{x \rightarrow \frac{\pi}{2}} (1 - \sin x) = 0 \quad \lim_{x \rightarrow \frac{\pi}{2}} (1 + \cos 2x) = 0$$

$$= 0/0$$

So by L'Hopital's rule

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \sin x}{1 + \cos 2x}$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{d/dx(1 - \sin x)}{d/dx(1 + \cos 2x)}$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{-\cos x}{-2 \sin 2x} = \frac{\cos \frac{\pi}{2}}{2 \sin \pi} = 0$$

Question No: 50 (Marks: 10)

$$\sum_1^{\infty} \frac{2^n}{n(n+2)}$$

Use the Ratio test to determine whether the series converges or diverges.

$$p = \lim_{x \rightarrow \infty} \frac{u_{k+1}}{u_k} =$$